International Journal of Technology, Science and Engineering **ISSN: 2457-1016**

Vol. 3, Issue II, Apr-Jun, 2020 http://www.bharatpublication.com/journal-detail.php?jID=25/IJTSE

On SÔZ Fuzzy-Ideal of KU-Algebra

*Showq Mohammed .E

**DhuhaAbdulameerKadhim

*,** *Iraq, University of Kufa, College of Education for Girls , Department of Mathematics*

ABSTRACT

In this paper, we present and study ideal in KU- Algebra, it is namedSÔZ-ideal ,we provide some examples, properties and theorems about it.Also,we study the direct product of SÔZ-ideals.Finally, we announce and study fuzzy SÔZ –ideal of this Algebra.

Keywords: *KU-algebra, fuzzy KU- ideal, σ -multiplication,fuzzy $\hbar$ -multiplication,onto homomorphism.*

1.INTRODUCTION

The perception of fuzzy subsets was defined viaZadeh in 1965 [8]. Then Iseki K.and Tanaka Sgaveconcept BCK& BCI-algebrasrespectively [1]. After that several papers have been available by mathematicians to defined the classical mathematical concepts and fuzzy mathematical concepts .Prabpayak C. &Leerawat UpresentedKU-Algebra. They offered the Definition ofKU homomorphisms and provided some related theories in [5]. In addition, Hameed A.T, Mostafa SM.et al. [4] acquaint with the fuzzy KU-ideals of aKU-Algebra. In this paper we itemized the ideas as we talk about in abstract.In the next parts of our research.

<u>Notice</u> We will symbolize to KU- Algebra $(\wp;\bullet,0)$ by $\wp$ and every fuzzy subset by F-set

The Basic Concepts

Definition 2.1: [8]

Let $\mu:\wp\rightarrow[0,1]$ bea F-set of nonempty set $\wp$.

Definition 2.2: [8]

Assume $\perp$ is a F-set of $\wp$. If $\perp(\tau\ell)=0$ for every $y\in\wp$ then $\perp$ is named empty F-set.

Definition 2.3: [8]

If $\perp$ and ∂ are two fuzzy sets. Then : $\forall\lambda\in\wp$.

1- $(\perp\bigcap\partial)(\lambda)=\text{MIN}\{\perp(\lambda),\partial(\lambda)\}$

2 – $(\perp\bigcup\partial)(\lambda)=\text{MAX}\{\perp(\lambda),\partial(\lambda)\}$.

Definition 2.4: [7]

The set $\wp$ is aKU-Algebra,with the constant and a binary operation $0,\bullet$ respectively if the four conditions are satisfying :-

$1-(\lambda \bullet \tau\ell)\bullet[(\tau\ell \bullet z))\bullet(\lambda \bullet z)=0$, $\forall \lambda, \tau\ell, z \in \wp$

$2-0\bullet \lambda=\lambda$, $\forall \lambda \in \wp$

$3-\lambda \bullet 0=0,$

$4-\lambda \bullet \tau\ell=0=\tau\ell \bullet \lambda \rightarrow \lambda=\tau\ell.$

Definition 2.5: [7]

We know $\perp$ is a fuzzy KU -ideal of $\wp$ when : $\forall \lambda, \tau\ell, z \in \wp.$

$1- \perp(0) \geq \perp(\lambda),$

$2- \perp(\lambda \bullet z) \geq \text{MIN}\{\perp((\lambda \bullet \tau\ell)\bullet z), \perp(\tau\ell)\}.$

Definition 2.6: [6]

Suppose that $\hbar \in (0,1]$ is a fuzzy $\hbar$ -multiplicationof χ define by $\chi_{\hbar}^{M}(\lambda)=\hbar\chi(\lambda); \forall \lambda \in \psi.$

Definition 2.7:[2]

Let χ be a F-set of $\wp$ and let $\sigma \in [0,T]$ .A map χ_{σ}^{T} from $\wp$ to [0,1]is known as a fuzzy translation of χ if it is fulfills $\chi_{\sigma}^{T}(\lambda)=\chi(\lambda)+\sigma$; $\forall \lambda \in \wp.$ Where T=1-sup{ $\chi(\lambda), \forall \lambda \in \wp$ }.

Theorem 2.8: [3]

Let $(\wp;\bullet,0)$ and $(G;\bullet',0')$ be two KU-Algebras and $\varpi:(\wp;\bullet,0)\rightarrow(G;\bullet',0')$ be an onto homomorphism . Then the image of any fuzzy Kuideal is also fuzzy Kuideal symbolized by $\varpi(\perp)$.

Definition 2.9: [2]

Let $\{\perp_{\varepsilon},\ \varepsilon \in \varpi\}$ be a set of F-set from $\wp$ .Define $\bigcap_{\varepsilon \in \varpi}\perp_{\varepsilon}(\lambda)=\inf_{\varepsilon \in \varpi}\{\perp_{\varepsilon}(\lambda)\}, \forall \lambda \in \wp$,

$\bigcup_{\varepsilon \in \varpi}\perp_{\varepsilon}(\lambda)=\sup_{\varepsilon \in \varpi}\{\perp_{\varepsilon}(\lambda), \forall \lambda \in \wp\}.$

Fuzzy SÔZ-ideal

Definition 3.1

A fuzzy ideal $\perp$ of $\wp$ is named a SÔZ fuzzy ideal and denoted it by SÔZ - F -ideal of $\wp$ if

$1- \perp(0) \geq \perp(\lambda);\ \ \forall \lambda \in \wp.$

$2- \perp(\lambda^{2} \bullet \tau\ell)) \geq \text{MIN}\{\perp(\lambda), \perp(\tau\ell)\}; \forall \lambda, \tau\ell \in \wp.$

Example 3.2

Let $\wp=\{0,\varepsilon,\tau,\partial\}$ be a set with the accompanying table

$\bullet$	0	ε	τ	∂	ℓ
0	0	ε	τ	∂	ℓ
ε	0	0	τ	∂	∂
τ	0	ε	0	ε	ℓ
∂	0	0	0	0	∂
ℓ	0	0	0	0	0

Then $(\wp,\bullet,0)$ is an KU -Algebra and defined F-set $\perp:\wp\to[0,1]$,when $\lambda\in[0,1],\lambda\succ\imath\succ\nu;\perp(0)=\lambda,\ \perp(\varepsilon)=\imath,\perp(\tau)=\imath,\perp(\partial)=\nu,\perp(\ell)=\nu$ is a SÔZ - F -ideal of $\wp$.

Noticewe will symbolize $\perp$ is SÔZ - F -ideal of $\wp$ by $\perp\nleftarrow\wp$.

Theorem 3.3

If $\perp_\varepsilon$ be a set of SÔZ - F -ideals of $\wp$, $\forall\varepsilon\in\angle$, then $\bigcap_{\varepsilon\in\angle}\perp_\varepsilon\nleftarrow\wp$.

Proof

$1-\perp_\varepsilon(0)\geq\ \perp_\varepsilon(\lambda);\forall\varepsilon\in\angle\ ,\ \ \forall\lambda,\tau\ell\in\wp.$

$$\bigcap_{\varepsilon\in\angle}\perp_\varepsilon(0)\geq\bigcap_{\varepsilon\in\angle}\perp_\varepsilon(\lambda).$$

$$2-\bigcap_{\varepsilon\in\angle}\perp_\varepsilon(\lambda^2\bullet\tau\ell)\geq\inf_{\varepsilon\in\angle}\{\perp_\varepsilon(\lambda^2\bullet\tau\ell)\}$$

$$=\inf_{\varepsilon\in\angle}\{\mathrm{MIN}\{\perp_\varepsilon(\lambda),\perp_\varepsilon(\tau\ell)\}\}$$

$$=\mathrm{MIN}\{\inf_{\varepsilon\in\angle}(\perp_\varepsilon(\lambda)),\inf_{\varepsilon\in\angle}(\perp_\varepsilon(\tau\ell))\}$$

$$=\mathrm{MIN}\{\bigcap_{\varepsilon\in\angle}\perp_\varepsilon(\lambda),\bigcap_{\varepsilon\in\angle}\perp_\varepsilon(\tau\ell)\}.$$

$$\Rightarrow\bigcap_{\varepsilon\in\angle}\perp_i\nleftarrow\wp.$$

Theorem 3.4

Let $\perp_\varepsilon$ be a chain of SÔZ - F -ideals of $\wp$, $\forall\varepsilon\in\angle$. Then $\bigcup_{\varepsilon\in\angle}\perp_\varepsilon\nleftarrow\wp.$

Proof

$1-\perp_\varepsilon(0)\geq\perp_\varepsilon(\lambda);\forall\varepsilon\in\angle,\forall\lambda,\tau\ell\in\wp.$

$$\bigcup_{\varepsilon\in\angle}\perp_\varepsilon(0)\geq\bigcup_{\varepsilon\in\angle}\perp_\varepsilon(\lambda).$$

$$2-\bigcup_{\varepsilon\in\angle}\perp_\varepsilon(\lambda^2\bullet\tau\ell)\geq\mathrm{SUP}_{\varepsilon\in\angle}\{\perp_\varepsilon(\lambda^2\bullet\tau\ell)\}$$

$$=\mathrm{SUP}_{\varepsilon\in\angle}\{\mathrm{MIN}\{\perp_\varepsilon(\lambda),\perp_\varepsilon(\tau\ell)\}\}$$

$$=\mathrm{MIN}\{\mathrm{SUP}_{\varepsilon\in\angle}(\perp_\varepsilon(\lambda)),\mathrm{SUP}_{\varepsilon\in\angle}(\perp_\varepsilon(\tau\ell))\}$$

$$=\mathrm{MIN}\{\bigcup_{\varepsilon\in\angle}\perp_\varepsilon(\lambda),\bigcup_{\varepsilon\in\angle}\perp_\varepsilon(\tau\ell)\}.$$

$\Rightarrow \bigcup_{\varepsilon \in \angle} \perp_{\varepsilon} \nrightarrow \wp.$

Proposition3.5

Let $\perp$ be a SÔZ - F -ideal of $\wp$. Then $\neg\perp(\lambda) = \{\lambda, \perp(\lambda), 1-\perp(\lambda)\}$ is alsoSÔZ - F -ideal of $\wp$.

Proof

1–

$\neg\perp(0) = \{0, \perp(0), 1-\perp(0)\} \geq \{\lambda, \perp(\lambda), 1-\perp(\lambda)\} = \neg\perp(\lambda).$

$2-\ \neg\perp(\lambda^2 \bullet \tau\ell) = \{\lambda^2 \bullet \tau\ell, \perp(\lambda^2 \bullet \tau\ell), 1-\perp(\lambda^2 \bullet \tau\ell)\} \geq \{\lambda^2 \bullet \tau\ell, \text{MIN}\{\perp(\lambda), \perp(\tau\ell)\}, 1-\text{Min}\{\perp(\lambda), \perp(\tau\ell)\}\}$

$= \{\lambda^2 \bullet \tau\ell, \text{MIN}\{\perp(\lambda), \perp(\tau\ell)\}, \text{MIN}\{1-\perp(\lambda), 1-\perp(\tau\ell)\}\} = \{\lambda^2 \bullet \tau\ell, \text{MIN}\{\perp(\lambda), \perp(\tau\ell), 1-\perp(\lambda), 1-\perp(\tau\ell)\}\}$

$= \{\lambda^2 \bullet \tau\ell, \text{MIN}\{\perp(\lambda), 1-\perp(\lambda)\}, \text{MIN}\{\perp(\tau\ell), 1-\perp(\tau\ell)\}\} = \{\lambda^2 \bullet \tau\ell, \neg\perp(\lambda), \neg\perp(\tau\ell)\}.$

$\Rightarrow \neg\perp(\lambda) \nrightarrow \wp.$

Proposition3.6

Let $\perp, \eta$ be two SÔZ - F -ideals of $\wp$. Then $\perp\times\eta(\lambda) = \text{MIN}\{\perp(\lambda), \eta(\lambda)\}$ is alsoSÔZ - F -ideal of $\wp$.

Proof

$1-\perp\times\eta(0) = \text{Min}\{\perp(0), \eta(0)\} \geq \text{Min}\{\perp(\lambda); \eta(\lambda)\} = \perp\times\eta(\lambda).$

$2-\perp\times\eta(\lambda^2 \bullet \tau\ell) = \text{MIN}\{\perp(\lambda^2 \bullet \tau\ell), \eta(\lambda^2 \bullet \tau\ell)\} \geq \text{MIN}\{\text{MIN}\{\perp(\lambda), \perp(\tau\ell)\}, \text{MIN}\{\eta(\lambda), \eta(\tau\ell)\}\}$

$= \text{MIN}\{\perp(\lambda), \eta(\lambda), \perp(\tau\ell), \eta(\tau\ell)\} = \text{MIN}\{\text{MIN}\{\perp(\lambda), \eta(\lambda)\}, \text{MIN}\{\perp(\tau\ell), \eta(\tau\ell)\}\}$

$= \text{MIN}\{\perp\times\eta(\lambda), \perp\times\eta(\tau\ell)\}.$

This means $\perp\times\eta \nrightarrow \wp.$

Theorem 3.7

Let $\perp \nrightarrow \wp \leftrightarrow \forall\sigma \in (0,1)$ $\perp_{\sigma}^{M}$ is σ -multiplication of SÔZ - F -ideal of $\wp$.

Proof: $\rightarrow$

Let $\sigma \in (0,1)$ buch that $\perp_{\sigma}^{M}$ is σ -multiplication ofSÔZ - F -ideal of $\wp$,

$1-\perp_{\sigma}^{M}(0) = \perp(0) \bullet \sigma \geq \perp(\lambda) \bullet \sigma = \perp_{\sigma}^{M}(\lambda)$

$2-\perp_{\sigma}^{M}(\lambda^2 \bullet y) \geq \perp(\lambda^2 \bullet y) \bullet \sigma \geq \text{MIN}\{\perp(\lambda), \perp(\tau\ell)\} \bullet \sigma = \text{MIN}\{\perp(\lambda) \bullet \sigma ; \perp(\tau\ell) \bullet \sigma\}$

$= \text{MIN}\{\perp_{\sigma}^{M}(\lambda); \perp_{\sigma}^{M}(\tau\ell)\}.$

From 1 and 2 $\perp_{\sigma}^{M}$ is σ -multiplication ofSÔZ - F -ideal of $\wp$.

Proof: $\leftarrow$

$1-\perp_{\sigma}^{M}(0) \geq \perp_{\sigma}^{M}(\lambda)$, $\forall\lambda \in \wp$, $\perp(0) \bullet \sigma \geq \perp(\lambda) \bullet \sigma \Rightarrow \perp(0) \geq \perp(\lambda).$

$2-\ \perp_{\sigma}^{M}(\lambda^2 \bullet \tau\ell) \geq \text{MIN}\{\perp_{\sigma}^{M}(\lambda), \perp_{\sigma}^{M}(\tau\ell)\}$ obviously $= \perp(\lambda^2 \bullet \tau\ell) \bullet \sigma \geq \text{MIN}\{\perp(\lambda), \perp(\tau\ell)\} \bullet \sigma$

$= \perp(\lambda^2 \bullet \tau\ell) \geq \text{MIN}\{\perp(\lambda), \perp(\tau\ell)\}.$

From 1 and 2, we get $\perp \nrightarrow \wp.$

Theorem 3.8

Let $f:(\wp;\bullet,0)\to(\wp';\bullet',0')$ be an endomorphism , $\perp \nrightarrow \wp$. Then $(\perp_\sigma^M)_f$ is σ-multiplication SÔZ - F-ideals of $\wp'$. When $(\perp_\sigma^M)_f(\hat\lambda)=\perp_\sigma^M(f(\hat\lambda))$.

Proof

Let $\hat\lambda, \tau\ell\in\wp$, $f(\hat\lambda)$, $f(\tau\ell)\in\wp'$.

$1-(\perp_\sigma^M)_f(0)=\perp_\sigma^M(f(0))=\perp_\sigma^M(0')=\sigma\bullet\perp(0')\ge\sigma\bullet\perp(f(\hat\lambda))=\perp_\sigma^M(f(\hat\lambda))=(\perp_\sigma^M)_f(\hat\lambda)$

$\Rightarrow(\perp_\sigma^M)_f(0)\ge(\perp_\sigma^M)_f(\hat\lambda)$.

$2-(\perp_\sigma^M)_f(\hat\lambda^2\bullet\tau\ell)=\perp_\sigma^M(f(\hat\lambda^2\bullet\tau\ell))\ge\sigma\bullet\perp(f(\hat\lambda^2\bullet\tau\ell))=\text{MIN}\{\sigma\bullet\perp(f(\hat\lambda)),\sigma\bullet\perp(f(\tau\ell))\}$

$=\text{MIN}\{\perp_\sigma^M(f(\hat\lambda)),\perp_\sigma^M(f(\tau\ell))\}=\text{MIN}\{(\perp_\sigma^M)_f(\hat\lambda),(\perp_\sigma^M)_f(\tau\ell)\}$.

$\Rightarrow(\perp_\sigma^M)_f$ is σ-multiplicationSÔZ - F -ideal of $\wp'$.

Theorem 3.9

If $\perp \nrightarrow \wp$, then fuzzy $\hat\lambda-$translation $\perp_\sigma^T$ of $\perp \nrightarrow \wp$ $\forall\sigma\in[0,\text{T}]$.

Proof

Assume $\perp \nrightarrow \wp$ and let $\sigma\in[0,\text{T}]$.Then

$1-\ \perp_\sigma^T(0)=\perp(0)+\sigma\ge\perp(\hat\lambda)+\sigma=\perp_\sigma^T(\hat\lambda)$.

$2-\perp_\sigma^T(\hat\lambda^2\bullet\tau\ell)=\perp(\hat\lambda^2\bullet\tau\ell)+\sigma\ge\text{Min}\{\perp(\hat\lambda),\perp(\tau\ell)\}+\sigma$

$\perp_\sigma^T(\text{x}^2\bullet\tau\ell)\ge\text{Min}\{\perp_\sigma^T(\text{x}),\perp_\sigma^T(\tau\ell)\}$.

From 1 and 2 , we have $\perp_{\hat\lambda}^T \nrightarrow \wp$.

Theorem 3.10

Let $\perp$ be fuzzy ideal of $\wp$.Such that a $\perp_\sigma^T \nrightarrow \wp$ Forsame $\sigma\in[0,\text{T}]$, then $\perp \nrightarrow \wp$.

Proof

Consider that $\perp_\sigma^T \nrightarrow \wp$ for some $\sigma\in[0,\text{T}]$.Then $\perp \nrightarrow \wp$.

$1-\perp(0)+\sigma=\perp_\sigma^T(0)\ge\perp_\sigma^T(\hat\lambda)=\perp(\hat\lambda)+\sigma$,

this leads $\perp(0)\ge\perp(\hat\lambda)$.

$2-\ \perp_\sigma^T(\hat\lambda^2\bullet\tau\ell)\ge\text{MIN}\{\perp_\sigma^T(\hat\lambda),\perp_\sigma^T(\tau\ell)\}=\text{MIN}\{\perp(\hat\lambda)+\sigma,\perp(\tau\ell)+\sigma\}=\perp(\hat\lambda^2\bullet\tau\ell)+\sigma$

$\ge\text{MIN}\{\perp(\hat\lambda);\perp(\tau\ell)\}+\sigma\Rightarrow\perp(\hat\lambda^2\bullet\tau\ell)\ge\text{MIN}\{\perp(\hat\lambda),\perp(\tau\ell)\}$

From 1 and 2,we have $\perp \nrightarrow \wp$.

REFERENCES

[1] K. Iseki &S.Tanaka, An introduction to the theory of BCK Algebras, Math, Japonica, pp.26-23, no.1,1978.

[2] V. Kumbhojkar , M. S. Bapat, Not-so-fuzzy ideals . Fuzzy Set and Systems, pp.237-

243,Vol.3(7).1991.

[3]B. Leek, Y.B. Jun and M.J. Doh,(),Fuzzy translations and fuzzy multiplications of BCK/BCL algebras, commumkorean math.,Vol-24,no.3 ,pp. 353-360, 2009.

[4] SM Mostafa, A.T. Hameed, NZ Mohammed, Fuzzy Translations of KUS-algebras, Journal of AL-Qadisiyah for computer science and mathematics, 8-16, 8 (2), 2016.

[5] C. Prabpayak , U. Leerawat, On isomorphism of KU Algebras, Scientia Magna., 25-31,Vol. 5(3). 2009.

[6] T. Priya and T. Ramachandran, fuzzy translation and Fuzzy multiplication on Ps-algebras. Inter. J. Innovation in Science and Mathematics, Vol. 2, no.5 , 485-489, 2014.

[7]M. Samy , Mostafa, Mokhtar A. Abd-Elnaby and Moustafa M. M. Yousef, Fuzzy Ideals of KU – Algebras , Int. Math. Forum , 3139-3149 ,Vol. 6(63), 2011.

[8] L.A. Zadeh, Fuzzy Set. Information and control, pp. 338-358, Vol. 8, 1965.